IMPLEMENTATION OF EFFICIENT INTEGER DCT ARCHITECTURES FOR HEVC USING VERILOG HDL

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Abstract: High Efficiency Video Coding (HEVC) inverse transform for residual coding uses 2-D 4x4 to 32x32 transforms with higher precision as compared to H.264/AVC’s 4x4 and 8x8 transforms resulting in an increased hardware complexity. In this paper, an energy and area-efficient VLSI architecture of an HEVC-compliant inverse transform and dequantization engine is presented. We implement a pipelining scheme to process all transform sizes at a minimum throughput of 2 pixel/cycle with zero-column skipping for improved throughput. We use data-gating in the 1-D Inverse Discrete Cosine Transform engine to improve energy-efficiency for smaller transform sizes. A high-density SRAM-based transpose memory is used for an area-efficient design. This design supports decoding of 4K Ultra-HD (3840x2160) video at 30 frame/sec. The inverse transform engine takes 98.1 kgate logic, 16.4 kbit SRAM and 10.82 pJ/pixel while the dequantization engine takes 27.7 kgate logic, 8.2 kbit SRAM and 1.10 pJ/pixel in 40 nm CMOS technology. Although larger transforms require more computation per coefficient, they typically contain a smaller proportion of non-zero coefficients. Due to this trade-off, larger transforms can be more energy-efficient.

Keywords- HEVC, Inverse Discrete Cosine Transform, Transpose Memory, Data Gating.

I. Introduction

The High Efficiency Video Coding (HEVC) standard is the most recent joint video project of the ITU-T Video Coding Experts Group (VCEG) and the ISO/IEC Moving Picture Experts Group (MPEG) standardization organizations, working together in a partnership known as the Joint Collaborative Team on Video Coding (JCT-VC) [1]. The first edition of the HEVC standard is expected to be finalized in January 2013, resulting in an aligned text that will be published by both ITU-T and ISO/IEC. Additional work is planned to extend the standard to support several additional application scenarios, including extended-range uses with enhanced precision and color format support, scalable video coding, and 3-D/stereo/multiview video coding. In ISO/IEC, the HEVC standard will become MPEG-H Part 2 (ISO/IEC 23008-2) and in ITU-T it is likely to become ITU-T Recommendation H.265.

Video coding standards have evolved primarily through the development of the well-known ITU-T and ISO/IEC standards. The ITU-T produced H.261 [2] and H.263 [3], ISO/IEC produced MPEG-1 [4] and MPEG-4 Visual [5], and the two organizations jointly produced the H.262/MPEG-2 Video [6] and H.264/MPEG-4 Advanced Video Coding (AVC) [7] standards. The two standards that were jointly produced have had a particularly strong impact and have found their way into a wide variety of products that are increasingly prevalent in our daily lives. Throughout this evolution, continued efforts have been made
to maximize compression capability and improve other characteristics such as data loss robustness, while considering the computational resources that were practical for use in products at the time of anticipated deployment of each standard.

The Discrete cosine transform (DCT) plays a vital role in video compression due to its near-optimal de-correlation efficiency [1]. Several variations of integer DCT have been suggested in the last two decades to reduce the computational complexity. The new H.265/High Efficiency Video Coding (HEVC) standard has been recently finalized and poised to replace H.264/AVC [8]. Some hardware architectures for the integer DCT for HEVC have also been proposed for its real-time implementation. Ahmed et al. [9] decomposed the DCT matrices into sparse sub-matrices where the multiplications are avoided by using the lifting scheme. Shen et al. used the multiplier less multiple constant multiplication (MCM) approach for four-point and eight-point DCT, and have used the normal multipliers with sharing techniques for 16 and 32-point DCTs. Park et al. [11] have used Chen’s factorization of DCT where the butterfly operation has been implemented by the processing element with only shifters, adders, and multiplexors. Budagavi and Sze [12] proposed a unified structure to be used for forward as well as inverse transform after the matrix decomposition.

One key feature of HEVC is that it supports DCT of different sizes such as 4, 8, 16, and 32. Therefore, the hardware architecture should be flexible enough for the computation of DCT of any of these lengths. The existing designs for conventional DCT based on constant matrix multiplication (CMM) and MCM can provide optimal solutions for the computation of any of these lengths, but they are not reusable for any length to support the same throughput processing of DCT of different transform lengths. Considering this issue, we have analyzed the possible implementations of integer DCT for HEVC in the context of resource requirement and reusability, and based on that, we have derived the proposed algorithm for hardware implementation. We have designed scalable and reusable architectures for 1-D and 2-D integer DCTs for HEVC that could be reused for any of the prescribed lengths with the same throughput of processing irrespective of transform size.

II. HEVC Coding Design and Feature Highlights

The HEVC standard is designed to achieve multiple goals, including coding efficiency, ease of transport system integration and data loss resilience, as well as implementability using parallel processing architectures. The following subsections briefly describe the key elements of the design by which these goals are achieved, and the typical encoder operation that would generate a valid bit stream.

A. Video Coding Layer

The video coding layer of HEVC employs the same hybrid approach (inter/intrapicture prediction and 2-D transform coding) used in all video compression standards since H.261. Fig. 1 depicts the block diagram of a hybrid video encoder, which could create a bit stream conforming to the HEVC standard.

An encoding algorithm producing an HEVC compliant bit stream would typically proceed as follows. Each picture is split into block-shaped regions, with the exact block partitioning being conveyed to the decoder. The first picture of a video sequence (and the first picture at each clean random access point into a video sequence) is coded using only intra picture prediction (that uses some prediction of data spatially from region-to-
region within the same picture, but has no dependence on other pictures). For all remaining pictures of a sequence or between random access points, interpicture temporally predictive coding modes are typically used for most blocks. The encoding process for interpicture prediction consists of choosing motion data comprising the selected reference picture and motion vector (MV) to be applied for predicting the samples of each block. The encoder and decoder generate identical interpicture prediction signals by applying motion compensation (MC) using the MV and mode decision data, which are transmitted as side information.

The residual signal of the intra- or interpicture prediction, which is the difference between the original block and its prediction, is transformed by a linear spatial transform. The transform coefficients are then scaled, quantized, entropy coded, and transmitted together with the prediction information. The encoder duplicates the decoder processing loop (see gray-shaded boxes in Fig. 1) such that both will generate identical predictions for subsequent data. Therefore, the quantized transform coefficients are constructed by inverse scaling and are then inverse transformed to duplicate the decoded approximation of the residual signal. The residual is then added to the prediction, and the result of that addition may then be fed into one or two loop filters to smooth out artifacts induced by block-wise processing and quantization. The final picture representation (that is a duplicate of the output of the decoder) is stored in a decoded picture buffer to be used for the prediction of subsequent pictures. In general, the order of encoding or decoding processing of pictures often differs from the order in which they arrive from the source; necessitating a distinction between the decoding order (i.e., bit stream order) and the output order (i.e., display order) for a decoder.

Video material to be encoded by HEVC is generally expected to be input as progressive scan imagery (either due to the source video originating in that format or resulting from deinterlacing prior to encoding). No explicit coding features are present in the HEVC design to support the use of interlaced scanning, as interlaced scanning is no longer used for displays and is becoming substantially less common for distribution. However, a metadata syntax has been provided in HEVC to allow an encoder to indicate that interlace-scanned video has been sent by coding each field (i.e., the even or odd numbered lines of each video frame) of interlaced video as a separate picture or that it has been sent by coding each interlaced frame as an HEVC coded picture. This provides an efficient method of coding interlaced video without burdening decoders with a need to support a special decoding process for it.

![Fig. 1. Typical HEVC video encoder (with decoder modeling elements shaded in light gray).](image)

**III. Algorithm for Hardware Implementation of Integer DCT for HEVC:**

In the Joint Collaborative Team-Video Coding (JCT-VC), which manages the standardization of HEVC, Core Experiment 10 (CE10) studied the design of core
transforms over several meeting cycles. The eventual HEVC transform design involves coefficients of 8-bit size, but does not allow full factorization unlike other competing proposals. It however allows for both matrix multiplication and partial butterfly implementation. In this section, we have used the partial-butterfly algorithm for the computation of integer DCT along with its efficient algorithmic transformation for hardware implementation.

A. Key Features of Integer DCT for HEVC

The $N$-point integer DCT 1 for HEVC given by [14] can be computed by a partial butterfly approach using a $(N/2)$-point DCT and a matrix–vector product of $(N/2) \times (N/2)$ matrix with an $(N/2)$-point vector as

$$
\begin{bmatrix}
y(0) \\
y(2) \\
\vdots \\
y(N-4) \\
y(N-2)
\end{bmatrix} = C_{N/2}
\begin{bmatrix}
a(0) \\
a(1) \\
\vdots \\
a(N/2-2) \\
a(N/2-1)
\end{bmatrix}
$$

where

$$
A_i = x(i) + x(N - i - 1)
$$

$$
b_i = x(i) - x(N - i - 1)
$$

for $i=0,1,\cdots,N/2-1$. $X=[x(0),x(1),\cdots,x(N-1)]$ is the input vector and $Y=[y(0),y(1),\cdots,y(N-1)]$ is $N$-point DCT of $X$. $C_{N/2}$ is $(N/2)$-point integer DCT kernel matrix of size $(N/2) \times (N/2)$. $MN/2$ is also a matrix of size $(N/2) \times (N/2)$ and its $(i, j)$th entry is defined as

$$m_{N/2}^{i,j} = c^{2i+1,j}_N$$

for $0 \leq i, j \leq N/2 - 1$

Where $C^{2i+1,j}_N$ is the $(2i + 1,j)$th entry of the matrix $C_N$.

B. Hardware Oriented Algorithm

Direct implementation of (1) requires $N^2/4 + MUL_{N/2}$ multiplications, $N^2/4+N/2+ADD_{N/2}$ additions, and 2 shifts where $MUL_{N/2}$ and $ADD_{N/2}$ are the number of multiplications and additions/subtractions of $(N/2)$-point DCT, respectively.

Computation of (1) could be treated as a CMM problem [15]–[17]. Since the absolute values of the coefficients in all the rows and columns of matrix $M_{N/2}$ (1b) are identical, the CMM problem can be implemented as a set of $N/2$ MCMs that will result in a highly regular architecture and will have low-complexity implementation. The kernel matrices for four-, eight-, 16-, and 32-point integer DCT for HEVC are given in [14], and 4- and eightpoint integer DCT are represented, respectively, as

$$
C_4 =
\begin{bmatrix}
64 & 64 & 64 & 64 \\
83 & 36 & -36 & -36 \\
64 & -64 & -64 & 64 \\
36 & -83 & 83 & -36
\end{bmatrix}
$$
Based on (1) and (2), hardware oriented algorithms for DCT computation can be derived in three stages as in Table I. For 8-, 16-, and 32-point DCT, even indexed coefficients of \([y(0), y(2), y(4), \ldots, y(N-2)]\) are computed as 4-, 8-, and 16-point DCTs of \([a(0), a(1), a(2), \ldots, a(N/2-1)]\), respectively, according to (1a). In Table II, we have listed the arithmetic complexities of the reference algorithm and the MCM-based algorithm for four-, eight-, 16-, and 32-point DCT. Algorithms for Inverse DCT (IDCT) can also be derived in a similar way.

**Proposed Architectures for Integer DCT Computation:**

**A. Proposed Architecture for Four-Point Integer DCT:**

The proposed architecture for four-point integer DCT is shown in Fig. 1(a). It consists of an input adder unit (IAU), a shift-add unit (SAU), and an output adder unit (OAU). The IAU computes \(a(0), a(1), b(0), \text{ and } b(1)\) according to STAGE-1 of the algorithm as described in Table I. The computations of \(t_1,36\) and \(t_1,83\) are performed by two SAUs according to STAGE-2 of the algorithm. The computation of \(t_0,64\) and \(t_1,64\) does not consume any logic since the shift operations could be rewired in hardware. The structure of SAU is shown in Fig. 1(b). Outputs of the SAU are finally added by the OAU according to STAGE-3 of the algorithm.

**B. Proposed Architecture for Integer DCT of Length 8 and Higher Length DCTs:**

The generalized architecture for N-point integer DCT based on the proposed algorithm is shown in Fig. 2. It consists of four units, namely the IAU, (\(N/2\))-point integer DCT unit, SAU, and OAU. The IAU computes \(a(i)\) and \(b(i)\) for \(i = 0, 1, \ldots, N/2-1\) according to STAGE-1 of the algorithm of Section II-B. The SAU provides the result of multiplication of input sample with DCT.
coefficient by STAGE-2 of the algorithm. Finally, the OAU generates the output of DCT from a binary adder tree of log $2^N-1$ stages. Fig. 3(a)–(c), respectively, illustrates the structures of IAU, SAU, and OAU in the case of eight-point integer DCT. Four SAUs are required to compute $t_i,89, t_i,75, t_i,50,$ and $t_i,18$ for $i = 0, 1, 2,$ and $3$ according to STAGE-2 of the algorithm. The outputs of SAUs are finally added by two-stage adder tree according to STAGE-3 of the algorithm. Structures for 16- and 32-point integer DCT can also be obtained similarly.

Fig. 2. Proposed generalized architecture for integer DCT of lengths $N=8, 16,$ and 32.

Fig. 3. Proposed architecture of eight-point integer DCT and IDCT. (a) Structure of IAU. (b) Structure of SAU. (c) Structure of OAU

C. Reusable Architecture for Integer DCT

The proposed reusable architecture for the implementation of DCT of any of the prescribed lengths is shown in Fig. 4(a). There are two $(N/2)$-point DCT units in the structure. The input to one $(N/2)$-point DCT unit is fed through $(N/2)$ 2:1 MUXes that selects either $[a(0), ..., a(N/2-1)]$ or $[x(0), ..., x(N/2-1)]$, depending on whether it is used for $N$-point DCT computation or for the DCT of a lower size. The other $(N/2)$-point DCT unit takes the input $[x(N/2), ..., x(N-1)]$ when it is used for the computation of DCT of $N/2$ point or a lower size, otherwise, the input is reset by an array of $(N/2)$ AND gates to disable this $(N/2)$-point DCT unit. The output of this $(N/2)$-point DCT unit is multiplexed with that of the OAU, which is preceded by the SAUs and IAU of the structure. The NAND gates before IAU are used to disable the IAU, SAU, and OAU when the architecture is used to compute $(N/2)$-point DCT computation or a lower size. The input of the control unit, $m_N$ is used to decide the size of DCT computation. Specifically, for $N=32,m_{32}$ is a 2-bits signal that is set to $\{00\}, \{01\}, \{10\},$ and $\{11\}$ to compute four-, eight-, 16-, and 32-point DCT, respectively. The control unit generates sel 1 and sel 2, where sel 1 is used as control signals of NMUXes and input of NAND gates before IAU. sel 2 is
used as the inputm(N/2) to two lower size reusable integer DCT units in a recursive manner. The combinational logics for control units are shown in Fig. 4(b) and (c) for N= 16 and 32, respectively. ForN=8,m8 is a 1-bit signal that is used as sel 1 while sel 2 is not required since fourpoint DCT is the smallest DCT. The proposed structure can compute one 32-point DCT, two 16-point DCTs, four eightpoint DCTs, and eight four-point DCTs, while the throughput remains the same as 32 DCT coefficients per cycle irrespective of the desired transform size.

Fig. 4. Proposed reusable architecture of integer DCT. (a) Proposed reusable architecture for N= 8, 16, and 32. (b) Control unit for N= 16. (c) Control unit for N=32

We present here a folded architecture and a full-parallel architecture for the 2-D integer DCT, along with the necessary transposition buffer to match them without internal data movement.

A. Folded Structure for 2-D Integer DCT

The folded structure for the computation of (N×N)-point 2-D integer DCT is shown in Fig. 5(a). It consists of one N-point 1-D DCT module and a transposition buffer. The structure of the proposed 4×4 transposition buffer is shown in Fig. 5(b). It consists of 16 registers arranged in four rows and four columns. (N×N) transposition buffer can store N values in any one column of registers by enabling them by one of the enable signals ENi fori=0,1,…,N−1. One can select the content of one of the rows of registers through the MUXes. During the firstNsuccessive cycles, the DCT module receives the successive columns of (N×N) block of input for the computation of STAGE-1, and stores the intermediate results in the registers of successive columns in the transposition buffer. In the nextNcycles, contents of successive rows of the transposition buffer are selected by the MUXes and fed as input to the 1-D DCT module. NMUXes are used at the input of the 1-D DCT module to select either the columns from the input buffer (during the first Ncycles) or the rows from the transposition buffer (during the next Ncycles).
Fig. 5. Folded structure of (N×N)-point 2-D integer DCT. (a) Folded 2-D DCT architecture. (b) Structure of the transposition buffer for input size 4×4

B. Full-Parallel Structure for 2-D Integer DCT

The full-parallel structure for (N×N)-point 2-D integer DCT is shown in Fig. 6(a). It consists of two N-point 1-D DCT modules and a transposition buffer. The structure of the 4×4 transposition buffer for full-parallel structure is shown in Fig. 6(b). It consists of 16 register cells (RC) [shown in Fig. 6(c)] arranged in four rows and four columns. N×N transposition buffer can store N values in a cycle either row-wise or column-wise by selecting the inputs by the MUXes at the input of RCs. The output from RCs can also be collected either row-wise or column-wise. To read the output from the buffer, a number of (2N−1):1 MUXes [shown in Fig. 6(d)] are used, where outputs of the ith row and the ith column of RCs are fed as input to the ith MUX. For the first N successive cycles, the ith MUX provides output of N successive RCs on the ith row. In the next N successive cycles, the ith MUX provides output of N successive RCs on the ith column. By this arrangement, in the first N cycles, we can read the output of N successive columns of RCs and in the next N cycles, we can read the output of N successive rows of RCs. The transposition buffer in this case allows both read and write operations concurrently. If for the N cycles, results are read and stored column-wise now, then in the next N successive cycles, results are read and stored in the transposition buffer row-wise. The first 1-D DCT module receives the input column-wise from the input buffer. It computes a column of intermediate output and stores in the transposition buffer. The second 1-D DCT module receives the rows of the intermediate result from the transposition buffer and computes the rows of 2-D DCT output row-wise. Suppose that in the first N cycles, the intermediate results are stored column-wise and all the columns are filled with intermediately results, then in the next N cycles, contents of successive rows of the transposition buffer are selected by the MUXes and fed as input to the 1-D DCT module of the second stage. During this period, the output of the 1-D DCT module of first stage is stored row-wise. In the next N cycles, results are read and written column-wise. The alternating column-wise and row-wise read and write operations with the transposition buffer continues. The transposition buffer in this case introduces a pipeline latency of N cycles required to fill in the transposition buffer for the first time.
Fig. 6. Full-parallel structure of (N×N)-point 2-D integer DCT. (a) Fullparallel 2-D DCT architecture.

Pruned DCT Architecture:

Fig. 7(a) illustrates a typical hybrid video encoder, e.g., HEVC, and Fig. 7(b) shows the breakdown of the blocks in the shaded region in Fig. 7(a) for main profile, where the encoding and decoding chain involves DCT, quantization, dequantization, and IDCT based on the transform design proposed in [14]. As shown in Fig. 7(b), a data before and after the DCT/IDCT transform is constrained to have maximum 16 bits regardless of internal bit depth and DCT length. Therefore, scaling operation is required to retain the word length of intermediate data. In the main profile that supports only 8-bit samples, if bit truncations are not performed, the wordlength of the DCT output would be \( \log_2 2N + 6 \) bits more than that of the input to avoid overflow. The output wordlengths of the first and second forward transforms are scaled down to 16 bits by truncating least significant \( \log_2 N \) and \( \log_2 N + 6 \) bits, respectively, as shown in the figure. The resulting coefficients from the inverse transforms are also scaled down by the fixed scaling factor of 7 and 12. It should be noted that additional clipping of \( \log_2 N - 1 \) most significant bits is required to maintain 16 bits after the first inverse transform and subsequent scaling.

The scaling operation, however, could be integrated with the computation of the transform without significant impact on the coding result. The SAU includes several left-shift operations as shown in Figs. 1(b) and 3(b) whereas the scaling process is equivalent to performing the right shift. Therefore, by manipulating the shift operations in the SAU circuit, we can optimize the complexity of the proposed DCT structure.
Fig. 7. Encoding and decoding chain involving DCT in the HEVC codec. (a) Block diagram of a typical hybrid video encoder, e.g., HEVC. Note that the decoder contains a subset of the blocks in the encoder, except for the having entropy decoding instead of entropy coding. (b) Breakdown of the blocks in the shaded region in (a) for main profile.

Fig. 8(a) shows the dot diagram of the SAU to illustrate the pruning with an example of the second forward DCT of length 8. Each row of dot diagram contains 17 dots, which represents output of the IAU or its shifted form (for 16 bits of the input wordlength). The final sum without truncation should be 25 bits. But, we use only 16 bits in the final sum, and the remaining 9 bits are finally discarded. To reduce the computational complexity, some of the least significant bits (LSB) in the SAU [in the gray area in Fig. 8(a)] can be pruned. It is noted that the worst-case error by the pruning scheme occurs when all the truncated bits are one. In this case, the sum of truncated values amounts to 88, but it is only 17% of the weight of LSB of the final output, 2^9. Therefore, the impact of the proposed pruning on the final output is not significant. However, as we prune more bits, the truncation error increases rapidly. Fig. 8(b) shows the modified structure of the SAU after pruning.
IV. RESULTS AND DISCUSSIONS

A. Synthesis Results of 1-D Integer DCT

We have coded the architecture derived from the reference algorithm of as well as the proposed architectures for different transform lengths in VHDL, and synthesized by Synopsys Design Compiler using TSMC 90-nm General Purpose (GP) CMOS Library. The word length of input samples are chosen to be 16 bits. The area, computation time, and power consumption (at 100-MHz clock frequency). It is found that the proposed architecture involves nearly 14% less areadelay product (ADP) and 19% less energy per sample (EPS) compared to the direct implementation of reference algorithm, in average, for integer DCT of lengths 4, 8, 16, and 32. Additional 19% saving in ADP and 20% saving in EPS are also achieved by the pruning scheme with nearly the same throughput rate. The pruning scheme is more effective for higher length DCT since the percentage of total area occupied by the SAUs increases as DCT length increases, and hence more adders are affected by the pruning scheme.

B. Comparison With the Existing Architectures

We have named the proposed reusable integer DCT architecture before applying pruning as reusable architecture-1 and that after applying pruning as reusable architecture-2. The processing rate of the proposed integer DCT unit is 16 pixels per cycle considering 2-D folded structure since 2-D transform of 32×32 block can be obtained in 64 cycles. In order to support 8Kultrahigh definition (UHD) (7680×4320) at 30 frames/s and 4:2:0 YUV format that is one of the applications of HEVC [20], the proposed reusable architectures should work at the operating frequency faster than 94
MHz (7680×4320×30×1.5/16). The computation times of 5.56 ns and 5.27 ns for reusable architectures-1 and 2, respectively (obtained from the synthesis without any timing constraint) are enough for this application. Also, the computation time less than 5.358 ns is needed to support 8K UHD at 60 frames/s, which can be achieved by slight increase in silicon area when we synthesize the reusable architecture-1 with the desired timing constraint. Existing architectures for HEVC for N=32 in terms of gate count that is normalized by area of 2-inputNANDgate, maximum operating frequency, processing rate, throughput, and supporting video format. The proposed reusable architecture-2 requires larger area but offers much higher throughput. Also, the proposed architectures involve less gate counts, as well as higher throughput.

C. Synthesis Results of 2-D Integer DCT

We also synthesized the folded and full-parallel structures for 2-D integer DCT. We have listed total gate counts, processing rate, throughput, power consumption, and EPS in Table VII. We set the operational frequency to 187 MHz for both cases to support UHD at 60 frames/s. The 2-D full-parallel structure yields 32 samples in each cycle after initial latency of 32 cycles providing double the throughput of the folded structure. However, the full-parallel architecture consumes 1.69 times more power than the folded architecture since it has two 1-D DCT units and nearly the same complexity of transposition buffer while the throughput of full-parallel design is double the throughput of folded design. Thus, the full-parallel design involves 15.6% less EPS.

V. CONCLUSION

In this paper, we presented a very low-complexity DCT approximation obtained via pruning. The resulting approximate transform requires only 10 additions and possesses performance metrics comparable with state-of-the-art methods, including the recent architecture presented in [24]. By means of computational simulation, VLSI hardware realizations, and a full HECV implementation, we demonstrated the practical relevance of our method as an image and video codec.

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